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Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let e be the elasticity of the rigid line AD , DR a prolongation of AD . Let $AB=a$, $\angle DAB=\beta$, $\angle DBC=\delta$, $\angle RDC=\theta$. Then $\angle ADB=\delta-\beta$, $\tan \theta = e \tan(\delta-\beta)$. $\therefore \theta = \tan^{-1}[e \tan(\delta-\beta)]$, and is known.

Since θ is independent of the velocity of projection, AC/AB is independent of this velocity.

$$AD : a = \sin \delta : \sin(\delta-\beta). \quad \therefore AD = \frac{a \sin \delta}{\sin(\delta-\beta)}.$$

$$AC : AD = \sin \theta : \sin(\theta-\beta). \quad \therefore AC = \frac{AD \sin \theta}{\sin(\theta-\beta)} = \frac{a \sin \delta \sin \theta}{\sin(\delta-\beta) \sin(\theta-\beta)}.$$

$$\therefore \frac{AC}{a} = \frac{AC}{AB} = \frac{\sin \delta \sin \theta}{\sin(\delta-\beta) \sin(\theta-\beta)}.$$

AVERAGE AND PROBABILITY.

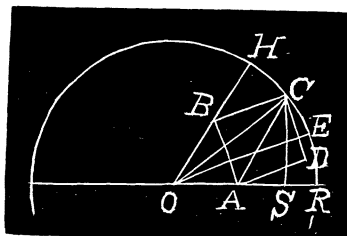
193. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

What is the average area of all squares that may be inscribed in a given sector of a circle, a diagonal of the square being parallel to a random line across the sector?

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

The following solution is for a sector less than a quadrant. Larger sectors would require a separate solution.

Let ROH be the given sector, $ABCD$ the inscribed square, C in the arc, A and B each in a radius; A in OR , B in OH ; AC the diagonal representing the random direction of the line; CS , perpendicular from C on OR . All directions will be included between AC in OR and AC parallel to the bisector of sector.



Let $AB=x$, $OC=OR=r$, $\angle HOR=\beta$, $\angle CAR=\theta$. Then $AC=x\sqrt{2}$, $AS=x\sqrt{2}\cos \theta$, $CS=x\sqrt{2}\sin \theta$, $OA=x\sin(\frac{1}{2}\pi+\theta-\beta)/\sin \beta$.

$$\therefore (x\sin(\frac{1}{2}\pi+\theta-\beta)/(\sin \beta+x\sqrt{2}\cos \theta))^2+2x^2\sin^2 \theta=r^2.$$

$$\therefore x^2 = \frac{r^2 \sin^2 \beta}{\sin^2(\frac{1}{2}\pi+\theta-\beta)+2\sqrt{2}\sin \beta \cos \theta \sin(\frac{1}{2}\pi+\theta-\beta)+2\sin^2 \beta}$$

$$= \frac{2r^2 \sin^2 \beta}{2+\sin 2\beta - \cos 2\beta + (1+\sin 2\beta)\sin 2\theta + (\cos 2\beta - 1)\cos 2\theta}.$$

The limits of θ are $\frac{1}{4}\pi$ and $\frac{1}{4}\pi + \frac{1}{2}\beta = \theta_1$.

$$\therefore \Delta = \frac{2}{\beta} \int_{\frac{1}{4}\pi}^{\theta_1} x^2 d\theta = \text{average area.}$$

$$\therefore \Delta = \frac{4r^2 \sin^2 \beta}{\beta(c-1)} \left[\tan^{-1} \left(\frac{c - \sqrt{(2c-1)}}{c-1} \tan \left(\frac{1}{4}\pi + \frac{1}{2}\beta - \frac{1}{2}\alpha \right) \right) - \tan^{-1} \left(\frac{c - \sqrt{(2c-1)}}{c-1} \tan \left(\frac{1}{4}\pi - \frac{1}{2}\alpha \right) \right) \right],$$

where $c = 2 + \sin 2\beta - \cos 2\beta$, $1 + \sin 2\beta = \sqrt{(2c-1)} \sin \alpha$,
 $\cos 2\beta - 1 = \sqrt{(2c-1)} \cos \alpha$.

$$\text{If } \beta = \frac{1}{4}\pi, \Delta = \frac{4r^2}{\pi} \{ \tan^{-1} \frac{1}{2} [\sqrt{10} - 3] [3 - \sqrt{5}] + \tan^{-1} \frac{1}{2} [3 - \sqrt{5}] [\sqrt{5} - 2] \}.$$

$$\therefore \Delta = .1933r^2.$$

If we accept the usual definition of inscription of geometric figures, viz., the vertices of the inscribed figure shall lie on the boundary of the other figure, the above problem is impossible; for a square cannot be inscribed in a sector so that its diagonal shall be parallel to a random direction. For a sector less than a quadrant three squares can be inscribed, and only three. The average area would be one-third of the sum of the areas of these three squares. The above solution satisfies the case when three vertices of the square lie on the boundary of the sector. The author of the problem may have wished the solution to cover the case when the vertices lie wholly within the sector, the limiting case being when three vertices lie on the boundary. In this case he should not have used the term "inscribed." ED. F.

MISCELLANEOUS.

172. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

If ϕ and ψ are small angles, show that an approximate value of ϕ/ψ is

$$\frac{\frac{2}{3} \sin \phi}{\frac{2}{3} \sin \psi} + \frac{\frac{1}{3} \tan \phi}{\frac{1}{3} \tan \psi} - \frac{1}{180} (\phi^2 - \psi^2) (9\phi^2 - \psi^2).$$

II. Solution by the PROPOSER.

Neither Prof. Zerr nor Mr. Greenwood seems to have carried the expansion quite far enough. When this is done the approximation stated will be found to be perfectly correct. The following relations will show this to be the case.

$$\begin{aligned} \frac{\frac{2}{3} \sin \phi}{\frac{2}{3} \sin \psi} &= \frac{2}{3} \left[\phi - \frac{\phi^3}{6} + \frac{\phi^5}{120} \right] \left[\psi - \frac{\psi^3}{6} + \frac{\psi^5}{120} \right]^{-1}, \text{ approximately,} \\ &= \frac{2}{3} \frac{\phi}{\psi} \left[1 - \left(\frac{\phi^2}{6} - \frac{\phi^4}{120} \right) \right] \left[1 - \left(\frac{\psi^2}{6} - \frac{\psi^4}{120} \right) \right]^{-1} \dots \end{aligned}$$